Progression through calculation methods for addition

These notes show the stages in building up to a compact, ***efficient*** method for addition. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage. The progression should be based on stage not age, i.e. progression should be based on ability regardless of year group. In all calculations children will be encouraged to approximate, using rounding and estimating to ensure that their answers are reasonable. Children should be encouraged to check their answers after calculation using an appropriate strategy.

Mental Calculations
Children need to be able to:

- know all number bonds to 10
- recall addition pairs to 9 + 9
- add mentally a series of single-digit numbers, such as 5 + 8 + 4
- count on in 1s, 10s and 100s
- use near doubles
- partition numbers into hundreds, tens and ones
- use the relationship between addition and subtraction
- partition numbers in ways other than into tens and ones to help with bridging multiples of 10 and 100
- add multiples of 10 or 100 (such as 60 + 70 or 600 + 700) using a related fact (6 + 7) **and knowledge of place value**
- mentally add multiples of 100, 10 and 1 e.g. 800 + 130 + 12

Many mental calculation strategies will continue to be used. They are not replaced by written methods.

Empty number line

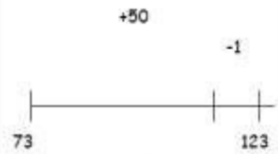
The empty number line helps to record the steps on the way to calculating the total.
The steps often bridge through a multiple of 10.

Example:
48 + 36 = 84

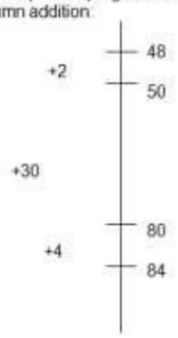
or:

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

49 + 73 = 122



Presenting number lines vertically will also help in the progression towards column addition.



Partitioning

When adding larger numbers, it becomes less efficient to count on so partitioning is used.
Partition into (hundreds) tens and ones, add to form partial sums and then recombine.

Partitioning all the numbers mirrors the standard column method where ones are placed under ones and tens under tens etc.

Example:
Partitioned numbers are written under one another:

47 + 76 = 40 + 7
= 70 + 6
110 + 13 = 123

375+567 = 300 + 70 + 5
500 + 60 + 7
800 + 130 + 12 = 942

Expanded column method

The expanded method leads children to the more compact column method so that they understand the structure and efficiency of it.

The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Example:

Initially the partitioning is still explicit:

67
+ 24
11 (7 + 4)
80 (60 + 20)
91

The partitioning is then done mentally:

Add the ones first
47
+ 76
13
110
123

The addition of the tens in the calculation 47 + 76 is described as 'Forty plus seventy equals one hundred and ten', stressing the link to the related fact 'Four plus seven equals eleven'.

Column method

The method is then shortened and when the column total is a two-digit number, the tens (or hundreds) are carried over into the next column. Use the words 'carry ten' or 'carry one hundred', **not** 'carry one'.

Example:

366
+ 458
824
1 1

Once learned, this method is quick and reliable. Later, extend to adding three two-digit numbers, two three-digit numbers, and numbers with different numbers of digits. This method can also be used to add decimals.

<u>**Progression through calculation methods for subtraction**</u>

These notes show the stages in building up to a compact, ***efficient*** method for subtraction. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage. The progression should be based on stage not age, i.e. progression should be based on ability regardless of year group. In all calculations children will be encouraged to approximate, using rounding and estimating to ensure that their answers are reasonable. Children should be encouraged to check their answers after calculation using an appropriate strategy.

Mental Calculations:

Children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact (16 – 7) and their knowledge of place value
- know all number bonds to 10 and 100
- find a small difference by counting up (e.g. 82 – 79 = 3
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one
- use the relationship between addition and subtraction
- partition numbers in different ways. e.g. 74 into 70 + 4 or 60 + 14
- subtract mentally a single-digit number or a multiple of 10 from a two-digit number
- add the totals (of the hundreds, tens and ones columns) mentally

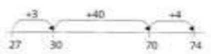
Many mental calculation strategies will continue to be used. They are not replaced by written methods.

Empty number line

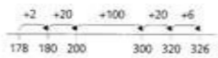
Empty or numbered lines are a useful way of modelling processes such as bridging through multiples of ten. The steps can be recorded by counting on or back.

Counting on example:

74 – 27 = 47



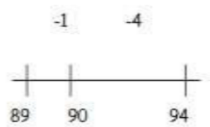
326 – 178 = 148



In some cases counting back will be a more efficient method of subtraction. Children should be shown both counting on and counting back, and be taught to select the **most efficient** method.

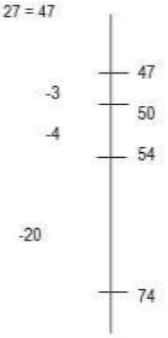
Counting back example:

94-5 = 89



Presenting number lines vertically will also help in the progression towards column addition:

74 – 27 = 47



The steps may be recorded in a different order or combined. With practice children will record less information and decide whether to count on or back.

Expanded column method

2 digit numbers with1 adjustment needed: 74 – 27 =

Expanded column method		Over time, recording is refined	Over time, recording is refined
Partition into tens and ones. Then say, "There is not enough to subtract 7 from 4", rather than, "You ***can't*** subtract 7 from 4".	70 + 4 is then readjusted to become 60 + 14. The calculation can now be carried out.	60 14 ~~70~~ + ~~4~~ – 20 + 7 40 + 7	6~~7~~1 4 –2 7 4 7 Say, "60 – 20" or, "6 tens – 2 tens" not, "6 – 4"
70 + 4 – 20 + 7	60 + 14 – 20 + 7 40 + 7		

3 digit numbers with1 adjustment needed: 563 – 271 =

Expanded column method		Over time, recording is refined	Over time, recording is refined
Partition into hundreds, tens and ones.	500 + 60 needs to be adjusted to become 400 + 160. The calculation can now be carried out.	400 160 ~~500~~ + ~~60~~ + 3 – 200 + 70 + 1 200 + 90 + 2	4~~5~~16 3 – 2 7 1 2 9 2 Say, "60 – 20" or, "6 tens – 2 tens" not, "6 – 4"
500 + 60 + 3 – 200 + 70 + 1	400 + 160 + 3 – 200 + 70 + 1 200 + 90 + 2		

3 digit numbers with 2 adjustments needed: 563 – 278 =

Expanded column method			Over time, recording is refined	Over time, recording is refined
This occurs when the tens ***and*** the ones to be subtracted are larger than those you are subtracting from	Firstly, 60+3 is adjusted to become 50+13	500+50 is then adjusted to become 400+150. The calculation can now be carried out.	150 400 ~~50~~ 13 ~~500~~ + ~~60~~ + ~~3~~ – 200 + 70 + 8 200 + 80 + 5	4~~5~~15~~6~~ 13 – 2 7 8 2 8 5 Say, "150 – 70" or "15 tens – 7 tens" not, "15 – 7"
500 + 60 + 3 – 200 + 70 + 8	500 + 50 + 13 – 200 + 70 + 8	400 + 150+ 13 – 200 + 70 + 8 200 + 80 + 5		

3 digit numbers with zeros where 2 adjustments are needed: 503 – 278 =

Expanded column method			Over time, recording is refined	Over time, recording is refined
When 0's are involved, the adjustments need to be done in a different order. There is not enough to subtract 8 from 3	As there are no tens, 500 + 0 is adjusted first to become 400 + 100	Then 100 + 3 can be adjusted to 90 + 13. The calculation can now be carried out.	90 13 400 ~~100~~ ~~500~~ + ~~0~~ + ~~3~~ – 200 + 70 + 8 200 + 20 + 5	4~~5~~9 1~~0~~13 – 2 7 8 2 2 5 Say, "100 – 70" or, "10 tens – 7 tens" not, "10 – 7" Once learned, this method is quick and reliable. It can be extended to subtracting larger numbers and numbers with different numbers of digits. It can also be used to subtract decimals.
500 + 0 + 3 – 200 + 70 + 8	400 + 100 + 3 – 200 + 70 + 8	400 + 90 + 13 – 200 + 70 + 8 200 + 20 + 5		

Progression through calculation methods for multiplication

These notes show the stages in building up to a compact, ***efficient*** method for multiplication. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage. The progression should be based on stage not age, i.e. progression should be based on ability regardless of year group. In all calculations children will be encouraged to approximate, using rounding and estimating to ensure that their answers are reasonable. Children should be encouraged to check their answers after calculation using an appropriate strategy.

Mental Calculations

Children need to be able to:

- count in steps
- understand multiplication as repeated addition
- partition numbers into multiples of one hundred, ten and one and in other ways
- recall multiplication facts to 10 × 10
- apply the knowledge of doubles and halves to known facts (e.g. 8x4 = double 4x4)
- work out products such as 70 × 5, 70 × 50, 700 × 5, or 700 × 50, using the related fact, 7 × 5, and an understanding of place value
- use the relationship between multiplication and division
- add combinations of numbers mentally or using a written method

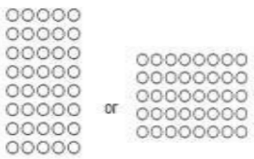
Many mental calculation strategies will continue to be used. They are not replaced by written methods.

Method 1 – Repeated addition

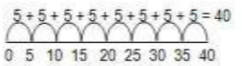
Children start by understanding multiplication as arrays and repeated addition. They use this understanding to help them work out multiplication facts they cannot recall quickly

Example:

For '8 x 5', children picture:



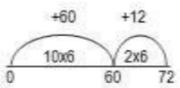
They use repeated addition to work out the calculation:



Recording of the steps on the number line may be refined as understanding and knowledge of facts develops.

Example:

12 x 6



This will support children in learning their tables using known facts and in understanding the distributive law which they will apply later when using the grid method.

Method 2 – Grid method into short/long multiplication

Multiplications can be carried out using the law of distribution which allows the numbers to be partitioned and each part to be multiplied separately. The products are then added to find the total product.

Stage 1 – Grid method

When multiplying a 1-digit number by a 2-digit number, children may choose to partition the numbers in different ways. Any of these 3 examples are acceptable, progressing towards the third, most efficient, example as knowledge of tables increases:

x	7
10	70
10	70
10	70
5	35
3	21
	266

x	7
10	70
10	70
10	70
8	56
	266

X	7
30	210
8	56
	266

Ensure that children understand the relationship between 7x3 and 7x30 and are not simply 'adding a nought'

The same method can also be applied when multiplying a 1-digit number by a 3-digit number:

x	6
500	3000
40	240
9	54
	3294

Ensure that children understand the relationship between 6x5 and 6x500 and are not simply 'adding 2 noughts'

When multiplying a 2-digit number by a 2-digit number:

(1) Partition both numbers and multiply each part

X	20	7
50	1000	350
6	120	42

(2) Add the answers in each row

X	20	7	
50	1000	350	1350
6	120	42	162

(3)Add the two row totals to find the final product

X	20	7	
50	1000	350	1350
6	120	42	162
			1512

Stage 2 – Short/long multiplication (only for most able and if appropriate)

```
  38
x  7
  56  (7 x 8)
 210  (7 x 30)
 266
```

Children describe what they are doing by referring to the value of the digits. Say, "30x7", not "3x7" although the relationship should be stressed

```
 549
x  6
  54
 240
3000
3294
```

Children say, "6x9, 6x40, 6x500". Once they are confident, children can omit the calculation in brackets.

```
  56
x 27
  42
 350
 120
1000
1512
```

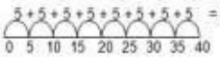
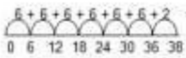
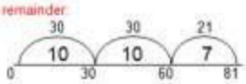
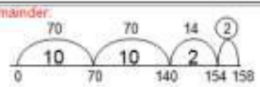
Children say, "7x6, 7x50, 20x6, 20x50"

All of these methods can also be used to multiply decimals.

<u>**Progression through calculation methods for division**</u>

These notes show the stages in building up to a compact, ***efficient*** method for division. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage. The progression should be based on stage not age, i.e. progression should be based on ability regardless of year group. In all calculations children will be encouraged to approximate, using rounding and estimating to ensure that their answers are reasonable. Children should be encouraged to check their answers after calculation using an appropriate strategy.

Mental Calculation	
Children need to be able to: • understand division as grouping and sharing • understand multiplication and division as inverse operations • recall multiplication and division facts to 10 × 10 • know how to find a remainder mentally, for example, find the remainder when 48 is divided by 5. Many mental calculation strategies will continue to be used. They are not	• derive larger multiples using known facts e.g. 10 x 3 = 30 → 20 x 3 = 60 etc • add multiples mentally and work out differences • partition two-digit and three-digit numbers into multiples of one hundred, ten and ones in different ways • recognise multiples of single-digit numbers • derive larger multiples using known facts replaced by written methods

Stage 1 - Repeated addition	Stage 2 – Vertical Chunking Method	Stage 3 – Long division
When it is not appropriate to use a sharing method for division and the division fact is not known, repeated addition (using the relationship between multiplication and division) can be used. Example without remainder: 40 ÷ 5 Ask "How many 5s in 40?"  = 8 fives Example with remainder: 38 ÷ 6  = 6 sixes with a remainder of 2 For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts. Example without remainder: 81 ÷ 3  This could either be done by working out the numbers of threes in each jump as you go along (10 threes are 30, another 10 threes makes 60, and another 7 threes makes 81. That's 27 threes altogether) or by counting in jumps of known multiples of 3 to reach 81 (30 + 30 + 21) then working out the number of threes in each jump.	72 ÷ 3 3) 72 − 30 10x 42 − 30 10x 12 − 6 2x 6 − 6 2x 0 Answer : 24 Any remainders should initially be shown as integers, progressing towards expressing them as fractions or decimals. Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context. e.g. I have 62p. Sweets are 8p each. How many can I buy? Answer: 7 (the remaining 6p is not enough to buy another sweet)	This method of HTU ÷ TU is an extension of the vertical chunking method. In the first example, the reasoning is made explicit on the right-hand side. 24) 560 20 − 480 24 × 20 80 3 72 24 × 3 8 Answer: 23 R 8 In the second example (below), the reasoning is implicit. 23 24) 560 −480 80 −72 8 Answer: 23 R 8
Example with remainder: 158 ÷ 7  10 sevens are 70, add another 10 sevens is 140, add 2 more sevens is 154 add 2 makes 158. So there are 22 sevens with a remainder of 2. The remainder is indicated above the jump rather than inside it, so that children do not mistakenly add 10, 10, 2 and 2 and get an answer of 24.	Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed? Answer: 8 (the remaining 6 apples still need to be placed into a box)	